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COMPARISON BETWEEN DIFFERENT MODELS FOR RHEOLOGICAL CHARACTERIZATION OF ACTIVATED SLUDGE

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ABSTRACT

Activated sludge flow rheology is a very complicated phenomenon. Studies related to activated sludge tend to classify sludge as non-Newtonian fluid. Until now, several theories have been built to describe the complex behavior of activated sludge with varying degrees of success. In this article, seven different models for viscosity of non-Newtonian fluids (i.e., Power law, Bingham plastic, Herschel-Bulkley, Casson, Sisko, Carreau and Cross) were considered to evaluate their predictive capability of apparent viscosity of activated sludge. Results showed that although evaluating the constants in the four-parameter models is difficult, they provide the best prediction of viscosity in the whole range of shear rates for activated sludge. For easier prediction of viscosity at different mixed liquor suspended solids (2.74-31g/L), temperature (15-25°C) and shear rate (1-1000/s), simple correlations were proposed. Comparing the results with the experimental data revealed that the proposed correlations are in good agreement with real apparent viscosities.

Key words: Activated sludge; Rheological characteristics; Viscosity models

INTRODUCTION

Since the second half of the twentieth century, the problem of water pollution has taken worrying proportions whereas, at the same time, water consumption increased together with the demographic explosion. In industrialized countries, the reduction and the control of water consumption is linked to the optimization of processes for industrial and domestic wastewater treatment. Advanced compact wastewater treatment processes are being looked for all over the world as effluent standards are becoming more stringent and land available for treatment plants more scarce (Mehrdadi *et al.*, 2006). The submerged membrane bioreactors (sMBR) process gives an advantage to other conventional activated sludge processes since it can treat high volumetric organic loadings by keeping a high concentration of sludge biomass (Yamamoto *et al.*, 1989; Cote *et al.*, 1998; Hassar *et al.*, 2004; Fallah *et al.*, 2010).

The operating temperature change of the MBR mainly affects the viscosity of the sludge and as a result it has a direct impact on membrane filtration behavior (Nagaoka, 1999; Meng *et al.*, 2006). Estimating the cost of energy related to the sludge circulation in the membrane and the shear stress at the filtration cake surface depends on the viscosity of the sludge in the reactor. More

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so, Viscosity has an impact on the rate of oxygen transfer, pressure loss in pipes, transport phenomena near the membrane and sludge conditioning (Chang *et al.*, 2002; Le-Clech *et al.*, 2006).

MBR process is more desirable due to high sludge concentration, which could be related to better oxidation, efficient pollutants removal, high quality effluent and reduced sludge yield in such a condition (Chang *et al.*, 2002; Le-Clech *et al.*, 2006). Aeration is the most significant parameter for controlling and prevention of fouling phenomena in MBR systems. However, it is also the most expensive component in terms of energy consumption. Viscosity is the parameter that can relate the aeration and mixed liquor suspended solids (MLSS) concentration.

Many researchers have studied the link between rheological properties of sludge flows and the parameters of the process in which the sludge is processed (Lolito *et al.*, 1997; Slatter, 1997; Mikkelson, 2001; Forster, 2002; Tixier *et al.*, 2003; Mori *et al.*, 2006; Mardani *et al.*, 2011). However, there is a lack of reliable literature data for viscosity of sludge in wastewater treatment systems such as activated sludge and membrane bioreactors mainly due to the nature of the systems under consideration (complex biological systems with non-Newtonian and often time-dependent behavior) inducing important space-time variations of sludge samples (Seyssiecq *et al.*, 2003). The purpose of this article was investigation of the viscosity of activated sludge in sMBR and establishing the appropriate model for it. Moreover, for easier prediction of viscosity at different mixed liquor suspended solids (MLSS), temperature and shear rate, some simple correlations were developed.

MATERIALS AND METHODS

Description of models

Shear stress (τ) is defined as the force required moving a given area of the fluid. The units of shear stress are Newtons/m^2, also known as Pascal. Shear rate (γ) is defined as the rate of movement of the fluid in proposed area, thus is measured in reciprocal seconds (1/s). Viscosity (η) is defined as the ratio of shear stress over shear rate. Consequently, the units are Newton seconds/m^2 or Pascal seconds. Another common unit of viscosity is Poise (dyne.s/cm^2).

Fig. 1 illustrates the viscosity versus shear rate curve for a non-Newtonian fluid. Of significance, there is usually a region at both low and high shear rates where viscosity is independent or nearly independent of shear rate and a section in between that exhibits strong shear rate dependence (Steffe, 1996).

Several mathematical models have been developed to describe the relationship between shear stress and shear rate of non-Newtonian fluids. These models are used to characterize flow properties to determine the ability of a fluid to perform specific functions. The most frequently applied models are the Power law (Ostwald) (Reiner, 1926) and Bingham plastic (Bingham, 1922) models, each of them has two adjustable parameters. The Power law model (Reiner, 1926) is expressed as:

$$\tau = K\gamma^n \qquad (1)$$

In the Power law model, the viscosity term from the Newtonian model is replaced with a constant, K, termed as the consistency index, which serves as a viscosity index of the system. The consistency index has the unusual set of units of Force-s^n/Area. In addition, the shear rate term is raised to the nth power, thus the term Power law. The factor, n, is called the flow behavior index which indicates the tendency of the fluid to shear thin.

The Bingham model (Bingham, 1922) is expressed as:

$$\tau = \tau_0 + \mu_p \gamma^n \qquad (2)$$

Fluids that exhibit Bingham plastic behavior are characterized by a yield point (τ_0) and a plastic viscosity (μ_p) that is independent of the shear rate.

However, these models are deficient in describing the overall rheological profile of activated sludge systems, especially in sMBR where the concentration of MLSS is high (Seyssiecq *et al.*, 2003). Furthermore, the presence of upper and lower Newtonian regions, coupled with a region of Power law behavior makes the interpretation and application of rheological data a challenging task.

There are several rheological models that involve three or more adjustable parameters. It is necessary to include a third parameter to describe the flow of the fluids in the upper or lower Newtonian region as well as the Power law region. The Herschel-Bulkley model corrects this deficiency by replacing the plastic viscosity term in the Bingham model with a Power law expression (Herschel and Bulkley, 1926) as follows:

$$\tau = \tau_0 + K\gamma^n \quad (3)$$

In addition the Casson equation is usually used to express the non-Newtonian behavior of activated sludge (Casson, 1956):

$$\tau - \tau_0^{1/2} = \eta_\infty^{1/2}\,\gamma^{1/2} \quad (4)$$

The Sisko model is another three parameter model, which is useful in describing flow in the Power law and upper Newtonian regions (Sisko, 1958).

$$\eta = \eta_\infty + K\gamma^n \quad (5)$$

There are also four parameter models that can be used over the entire range of shear rates. These models are mostly used for food and beverages (Steffe, 1996) and blood flow (Shibeshi *et al.*, 2005). Thus, it can be expected that these models have good capability in activated sludge flows. One of these models is Carreau model (Carreau, 1968):

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left(1 + (\lambda\gamma)^2\right)^{\frac{n-1}{2}} \quad (6)$$

Parameter λ is a time constant, calculated from the point on the viscosity versus shear rate curve where the flow changes from the lower Newtonian region to the Power law region. Cross model is similar to Carreau model in that it is a four parameter model that covers the entire shear rate range (Cross, 1965):

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{(1 + (\lambda\gamma)^m)} \quad (7)$$

In this equation, λ is the time constant and *m* is a dimensionless exponent. In general, the four parameter models are difficult to apply because there is seldom enough data to allow good model fitting. However, they represent the best results in predicting the behavior of non-Newtonian fluids like activated sludge.

According to Guibaud *et al.* (2004) and Baudez *et al.* (2004) the Herschel-Bulkley is proper to describe the viscosity of activated sludge at high concentrations; besides, the Bingham model is better for characterizing weakly concentrated sludge. Hasar *et al.* (2004), Laera *et al.* (2007) and Pollice *et al.* (2007) presented that the Power law model was the best for representing the viscosity of activated sludge in a sMBR, for a low shear rate range. Moreover, a complete review of the articles till 2002 can be found in Seyssiecq *et al.* (2003).

The models presented in the previous section were tested against the experimental data of Rosenberger *et al.* (2002) and Yang *et al.* (2009) to check their validity and determine the best rheological model for the sludge containing systems. These data were taken from MBRs which treated municipal wastes with submerged modules. MLSS concentration range was 20-31 g/L in Rosenberger *et al.* (2002) and 2.74-16 g/L in Yang *et al.* (2009).

RESULTS

Determination of model parameters

The models described in the previous section were fitted to the experimental data at various

Table1: Linear regression parameters for different rheological models with different MLSS concentrations.

Model	MLSS (g/L)	Parameters			R²
Power law		K	n		
	2. 74	1.71E-01	7.06E-02		9.83E-01
	5.08	6.87E-01	1.13E-02		9.89E-01
	7.43	6.58E-01	8.73E-02		9.68E-01
	10.22	9.37E-01	2.06E-01		9.79E-01
	16	1.75E+00	9.56E-02		9.97E-01
	20	9.21E+02	2.29E-01		9.91E-01
	26	1.08E+03	3.44E-01		9.89E-01
	31	5.74E+03	2.35E-02		9.96E-01
Bingham		τ_o	μ_p		
	2. 74	1.74E-01	6.68E-04		9.81E-01
	5.08	6.92E-01	4.52E-03		9.89E-01
	7.43	6.73E-01	2.36E-03		9.64E-01
	10.22	9.75E-01	1.76E-02		9.58E-01
	16	1.77E+00	2.33E-02		9.95E-01
	20	1.38E+03	1.01E+01		9.85E-01
	26	1.98E+03	2.12E+01		9.54E-01
	31	5.92E+03	1.12E+01		9.97E-01
Casson		τ_o	η_0		
	2. 74	1.64E-01	1.02E-02		9.81E-01
	5.08	7.17E-01	1.60E-02		9.89E-01
	7.43	6.26E-01	2.59E-02		9.66E-01
	10.22	7.99E-01	8.86E-02		9.69E-01
	16	1.59E+00	6.74E-02		9.97E-01
	20	1.07E+03	1.98E+00		9.92E-01
	26	1.38E+03	3.31E+00		9.77E-01
	31	5.57E+03	9.50E-01		9.96E-01
Herschel-Bulkley		τ_o	K	n	
	2. 74	6.03E+00	6.20E+00	2.33E-03	9.83E-01
	5.08	7.01E-01	1.15E-02	7.98E-01	9.89E-01
	7.43	2.75E+01	2.81E+01	2.50E-03	9.68E-01
	10.22	2.34E+02	2.35E+02	1.19E-03	9.84E-01
	16	4.75E+01	4.92E+01	3.76E-03	9.97E-01
	20	1.02E+03	1.38E+02	5.60E-01	9.92E-01
	26	8.31E+03	8.78E+03	8.82E-02	9.91E-01
	31	5.94E+03	8.05E+00	1.05E+00	9.97E-01

temperatures and MLSS concentrations by means of multiple non-linear regressions and the least square technique (Constantinides and Mostofi, 1999). The calculated parameters are listed in Table 1. The correlation coefficient (R^2) of the estimated η indicates that the Carreau model best fits the experimental data over the entire MLSS concentration range, followed by the Cross equation. Herschel-Bulkley and Casson models also fit the experimental data satisfactorily but are not as good as Carreau model.

Effect of MLSS concentration

The experimental data of Rosenberger *et al.* (2002) and Yang *et al.* (2009) were processed by multiple linear regression analysis, using the

Table1. Cont'd

Sisko		η_∞	K	n		R^2
	2. 74	1.33E-03	1.72E-01	9.19E-02		9.83E-01
	5.08	5.61E-03	6.91E-01	1.38E-02		9.89E-01
	7.43	8.21E-03	6.64E-01	1.21E-01		9.68E-01
	10.22	2.33E-02	9.53E-01	2.61E-01		9.90E-01
	16	3.53E-02	1.75E+00	8.97E-02		9.97E-01
	20	2.80E+00	3.27E+02	1.55E-01		9.92E-01
	26	4.38E+00	1.00E+03	1.76E-01		9.92E-01
	31	1.12E+00	1.06E+03	3.51E-01		9.89E-01
Carreau		η_0	η_∞	λ	n	
	2. 74	2.09E-01	2.49E-03	5.76E-01	7.56E-01	9.97E-01
	5.08	1.09E+00	1.83E-03	1.04E+00	4.30E-01	9.92E-01
	7.43	8.38E-01	5.69E-03	6.81E-01	5.45E-01	9.80E-01
	10.22	1.01E+00	8.30E-03	4.72E-01	5.76E-01	1.00E+00
	16	2.98E+00	1.94E-02	1.31E+00	3.02E-01	9.99E-01
	20	7.58E+00	4.38E-03	1.16E+01	1.70E-01	9.92E-01
	26	4.55E+00	7.08E-03	1.80E-01	1.79E-01	9.93E-01
	31	1.00E+01	1.58E-02	1.26E+01	2.82E-01	9.97E-01
Cross		η_0	η_∞	λ	m	
	2. 74	1.61E-01	1.61E-01	1.61E+00	1.61E+00	9.55E-01
	5.08	1.39E+00	2.43E-03	1.05E+00	1.41E+00	9.92E-01
	7.43	8.91E-01	7.28E-03	5.88E-01	1.67E+00	9.79E-01
	10.22	1.06E+00	9.89E-03	4.05E-01	1.71E+00	1.00E+00
	16	4.85E+00	2.09E-02	1.67E+00	1.17E+00	9.98E-01
	20	2.57E+00	4.39E-03	8.29E+02	8.25E-01	9.92E-01
	26	7.03E+00	8.46E-03	1.95E-01	9.69E-01	9.92E-01
	31	1.15E+01	1.58E-02	3.53E+02	1.06E+00	9.97E-01

least square technique. The relationship between viscosity, MLSS (ϕ_p (g/L)) and shear rate at 20 ºC can be given by:

$$\eta = 2.85 \times 10^{-2} \frac{\phi_p^{1.5}}{\gamma} \qquad (8)$$

The correlation coefficient for this equation was calculated to be 0.934. A comparison between the calculated apparent viscosity and experimental values is shown in Fig. 2. This figure demonstrates that Eq. (8) can predict the apparent viscosity better than the correlations of Rosenberger *et al.* (2002) and Yang *et al.* (2009). The derived correlation has a good prediction capability, especially at lower and upper Newtonian regions.

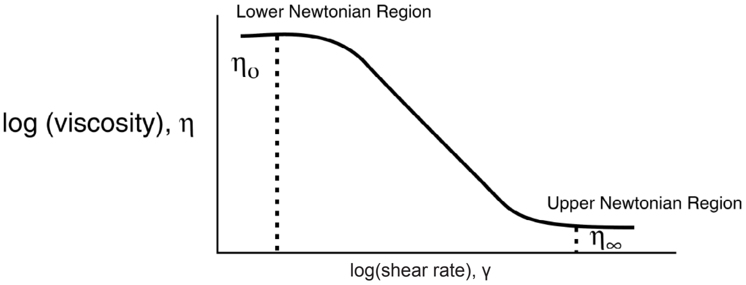

Fig. 1: Viscosity of a Non-Newtonian Fluid (Steffe, 1996).

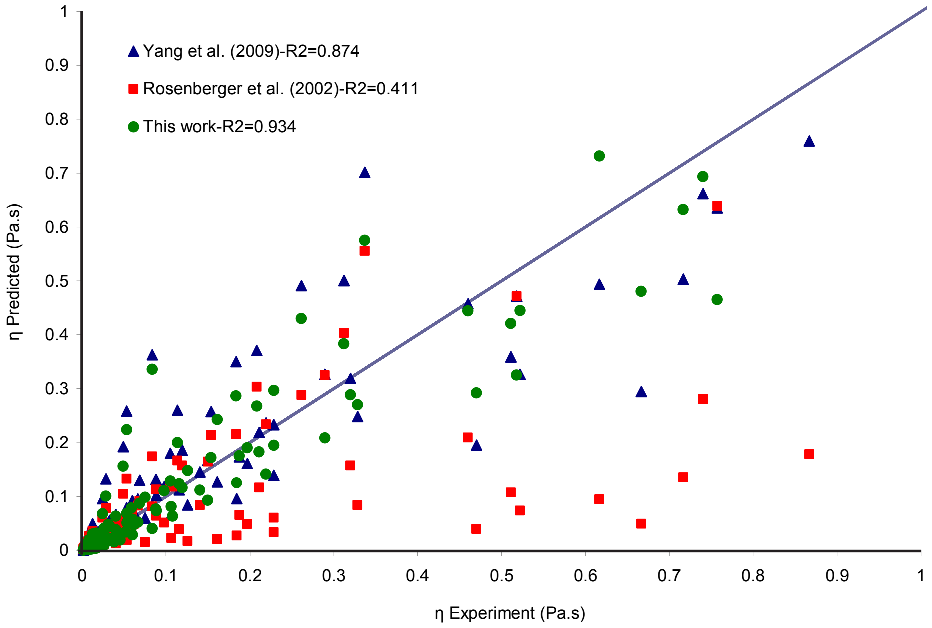

Fig. 2: Comparison of predicted viscosity by Eq. (8), Yang *et al*. (2009) and Rosenberger *et al*. (2002) with the experimental data.

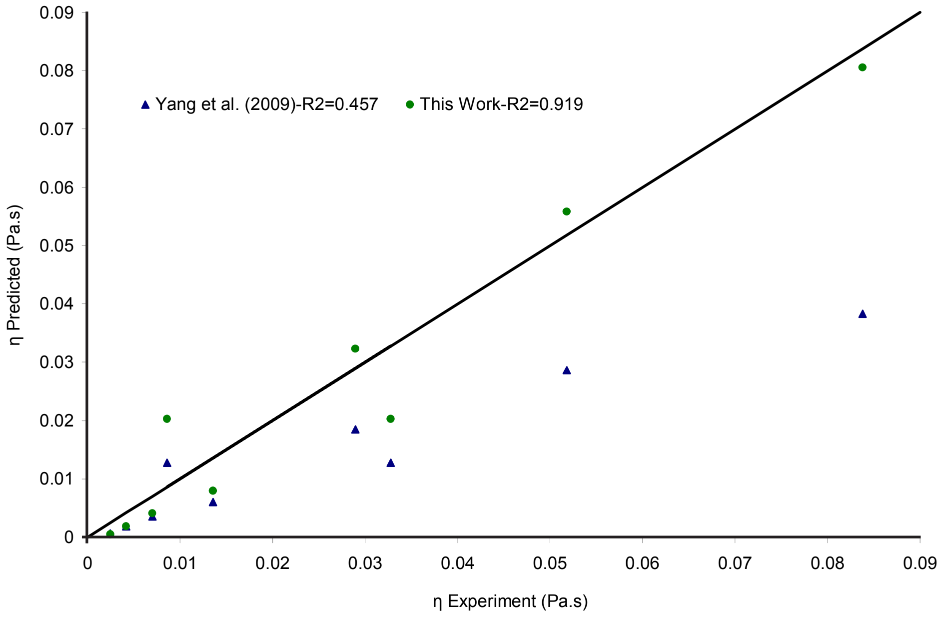

Fig. 3: Comparison of predicted data by Eq. (12) and Yang *et al*. (2009) with the experimental data

According to Popovic and Robinson (1984), the shear rate can be related to the aeration intensity (U_{Gr} (m/s)) as follows:

$$\gamma = 5000 U_{Gr} \tag{9}$$

Combining Eqs. (8) and (9) results in:

$$\eta = 5.7 \times 10^{-6} \frac{\phi_p^{1.5}}{U_{Gr}} \tag{10}$$

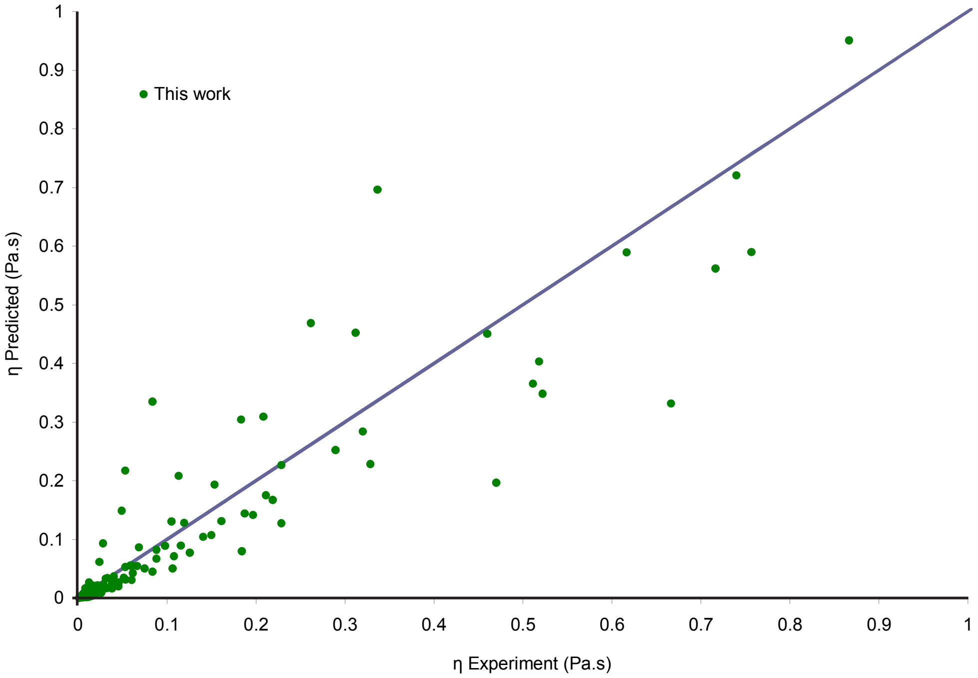

Fig. 4: Comparison of predicted data by Eq. (13) with the experimental data

This reveals that in higher aeration intensity due to increase in the shear rate, the viscosity of the activated sludge decreases.

Effect of temperature

Effect of temperature and MLSS concentration on viscosity at constant shear rate can be described as follows (Vitali and Rao, 1984):

$$\eta = k\phi_p^{\varepsilon} e^{\frac{E_a}{R_g.T}} \qquad (11)$$

where k and ε are dimensionless constants. E_a is activation energy for viscosity (KJ/mol), R_g is universal gas constant (8.3145 J/ºK.mol) and T is temperature in ºK. Using the experimental data of Rosenberger *et al.* (2002) and Yang *et al.* (2009), the constants of this equation were calculated as follows:

$$\eta = 4.05 \times 10^{-4} \phi_p^{1.5} e^{-7.7/Rg.T} \qquad (12)$$

This correlation was derived for constant shear rate of 100/s. The parity plot of calculated viscosity from Eq. (12) against the experimental values is shown in Fig. 3 and the corresponding correlation coefficient was calculated to be 0.919. Results of the correlation of Yang *et al.* (2009) are also presented in the same figure. This figure indicates that the correlation derived in this work is in good agreement with the experimental data and provides better prediction than that of Yang *et al.* (2009) in the whole range of MLSS concentration considered in this work.

A more generalized correlation was developed by considering the effect of shear rate on the apparent viscosity:

$$\eta = 2.86 \times 10^{-2} \frac{\phi_p^{1.5}}{U_{Gr}} \frac{-7.7}{Rg.T} \qquad (13)$$

The correlation coefficient for this equation was calculated to be 0.935. Comparison between the predicted viscosities by Eq. (13) and the experimental data can be seen in Fig. 4. This equation has good predictability for the data used in the whole ranges of MLSS, shear rate and temperature.

DISCUSSION

Seven different models for viscosity of non-Newtonian fluids (i.e., Power law, Bingham plastic, Herschel-Bulkley, Casson, Sisko, Carreau and Cross) were considered to evaluate their predictive capability of apparent viscosity of activated sludge in order to obtain a deeper understanding of the activated sludge structure and flow characteristics (Table 1). The results indicated that the Carreau model provides the best prediction of viscosity over the entire MLSS concentration range, followed by the Cross model. The Herschel-Bulkley and Casson models are also suitable but with less accuracy.

Herschel–Bulkley model covers all possible shear rates and assumes that the flow is homogeneous. This is provided that the stress is homogeneous. However, at low shear rates, the shear localization phenomena occurs, which means that the shear localizes in a small region with high local shear rate while the remaining part of the fluid behaves like a solid (Pignon *et al*., 1996; Pignon *et al*., 1996; Britton and Callaghan, 1997; Coussot *et al*., 2002; Møller *et al*., 2006). Although this effect is well-known to those who tried to make the activated sludge, or other similar suspensions, flow in a homogeneous fashion, this problem has received much less attention. Moreover, it can be observed in Table 1 that the Bingham equation is also suitable at low MLSS concentrations (MLSS < 10 g/L).

The results shown in Table 1 reveal that by increasing the MLSS concentration, the behavior of the fluid becomes more non-Newtonian.

Three correlations were proposed to relate the apparent viscosity to MLSS concentration, shear rate and temperature. MLSS concentration, temperature and shear rate obviously influence the flow characteristics of the activated sludge. The viscosity of the sludge increases exponentially with the increase of the MLSS concentration and decreases exponentially with increasing the temperature and the shear rate. The changes of the viscosity with MLSS concentration and shear rate were considerably higher than that with temperature.

The figs. 2-4 show that the models given in this work are in good agreement with experimental data and provide better prediction than the correlations of Rosenberger *et al.* (2002) and Yang *et al.* (2009) in the whole range of parameters (MLSS (2.74-31g/L), temperature (15-25°C) and shear rate (1-1000/s)). These models are especially useful in hydrodynamic optimization and also computational fluid dynamic (CFD) simulation of airlift and other kinds of MBRs and activated sludge processes, where viscosity plays a major role on the hydrodynamic regime, oxygen transfer and mass transport and consequently influences the system performance (Khalili-Garakani *et al*., 2011).

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